Kuwait University Math.101 Date : July 25, 1996.
Dept. of Maths. & Comp. Sc. Second Exam. Duration : 75 minutes.

Calculators are not allowed

Answer the following questions:

(7 points) Use the differentials to find an approximate value of $(4.02)^3 + 4\sqrt{4.02}$.

(7 points) Find an equation for the tangent line to the graph of

$$1 + 16x^2y = \tan(x - 2y) + 4y$$

at the point $\left(\frac{\pi}{4}, 0\right)$

(7 points) A right circular cone whose height h, is twice the radius of its base r. If the height of the cone increases at a rate of $\frac{dh}{dt} = 4\ cm/sec$, then find the rate of change of the volume of the cone when $r = 2\ cm$.

(2+5 points) State Rolle's theorem. Let: $f(x) = \cos\frac{x}{2}$, find all real numbers $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfy the conclusion of Rolle's theorem for f.

Let

$$f(x) = \frac{x}{\sqrt{x^2 - 9}}.$$

(a) (2 points) Show that $f''(x) = \frac{27x}{(x^2-9)^{\frac{5}{2}}}$, for every x in the domain of f.

(b) (2 points) Find the intervals on which f is increasing or decreasing, and find the local extrema of f (if any).

(c) (2 points) Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any).

(d) (3 points) Find the vertical and horizontal asymptotes for the graph of f (if any).

(e) (3 points) Sketch the graph of f.

(Good Luck)